2'nd order filter ID from Bode Plot

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The extraction technique depends on what system you assume to be extracting.

- Low Pass Filter
  - Order (2) $H(s) = \frac{\omega_0 G_{DC}}{\mathrm{s}+\omega_0}, \frac{\omega_0^N G_{DC}}{(\mathrm{s}+\omega_0)^N}, \frac{\omega_0^2 G_{DC}}{s^2+2\zeta\omega_0 s+\omega_0^2}$
- High Pass Filter
  - Order (2) $H(s) = \frac{s G_{DC}}{\mathrm{s}+\omega_0}, \frac{s^N G_{AC}}{(\mathrm{s}+\omega_0)^N}, \frac{s^2 G_{AC}}{s^2+2\zeta\omega_0 s+\omega_0^2}$
- Band Pass
  - Order (2)$H(s) = \frac{s 2\omega_0 \zeta G_{MB}}{s^2+2\zeta\omega_0 s+\omega_0^2}$
- Notch (Band Stop)
  - Order (2) $H(s) = \frac{s^2+\omega_o^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
- None of the above? (Any system not just filters have transfer functions.)
  - $H(s) = \frac{(s+1)\times 10\times\omega_0^2}{(s+10)\times(s^2+2\zeta\omega_0 s+\omega_0^2)}$

First order Low Pass

- Single pole has to bee real, as complex poles come in pairs $(\pm\sqrt{b^2 = 4ac})$
- $H(s) = \frac{\omega_0 G_{DC}}{s+\omega_0}$

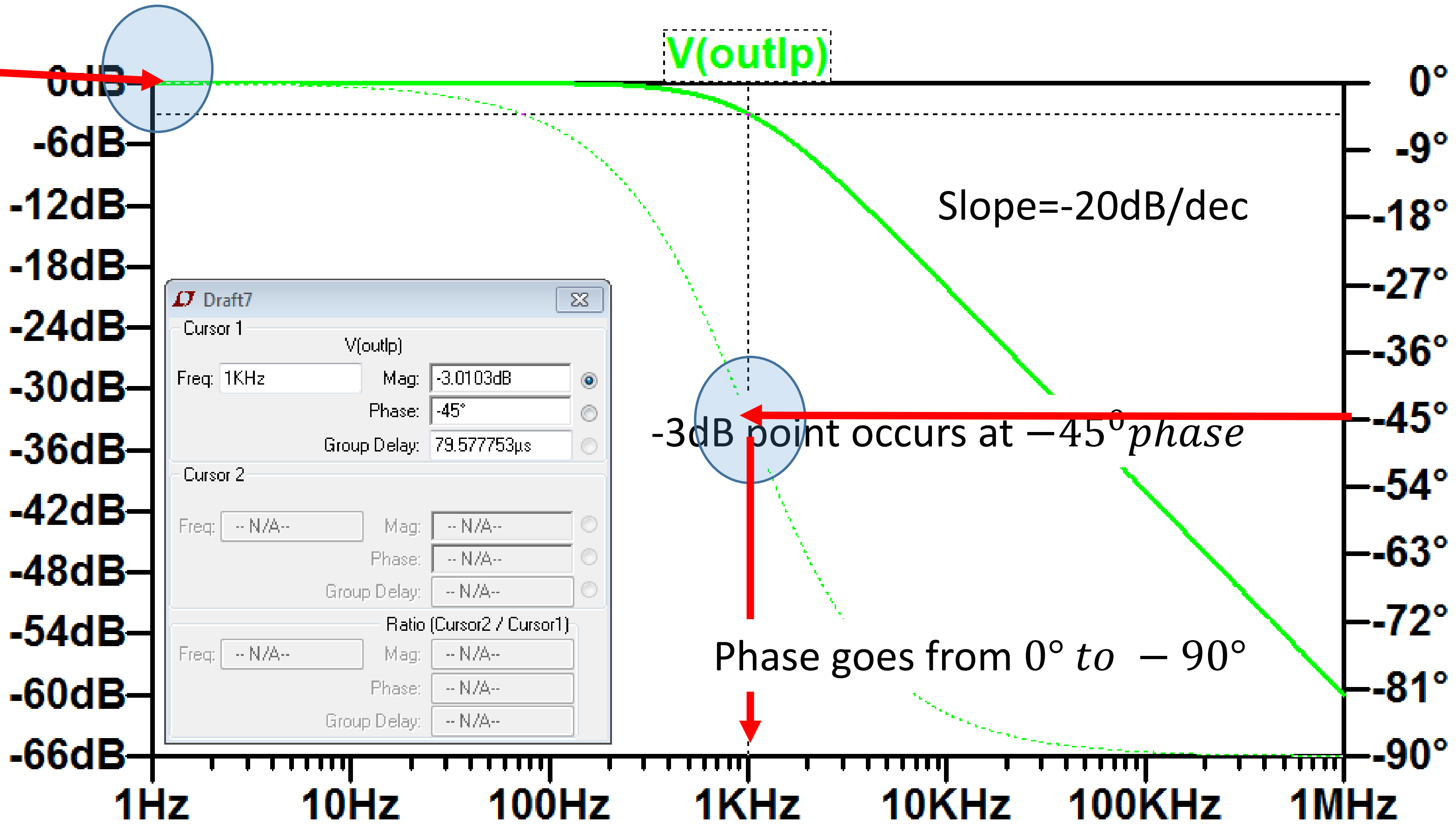
To find $\omega_0, set\ H(s = j\omega_0) = \frac{\omega_0 G_{DC}}{j\omega_0+\omega_0} = \frac{G_{DC}}{j+1} = \frac{G_{DC}}{\sqrt{2}\angle 45°} = \frac{1}{\sqrt{2}}\angle - 45° = -3dB\angle - 45°$

To find G_{DC} set s=0. $H(0) = \frac{\omega_0 G_{DC}}{0+\omega_0} = G_{DC}$

$$G_{DC} = 0dB = 10^{\frac{0}{20}} = 1$$

For a first order low pass filter
$\omega_0\ can\ be\ found\ at\ -45°\ phase$
$$\omega_0 = 2 \times pi \times 1kHz$$
We can then make sure the gain is -3dB down from G_{DC}.

$$H(s) = \frac{2 \times \pi \times 1k \times 1}{s + 2 \times \pi \times 1k}$$



Ltspice File

First order High Pass

- Single pole has to real, as complex poles come in pairs $(\pm\sqrt{b^2 = 4ac})$
- $H(s) = \frac{sG_{AC}}{s+\omega_0}$

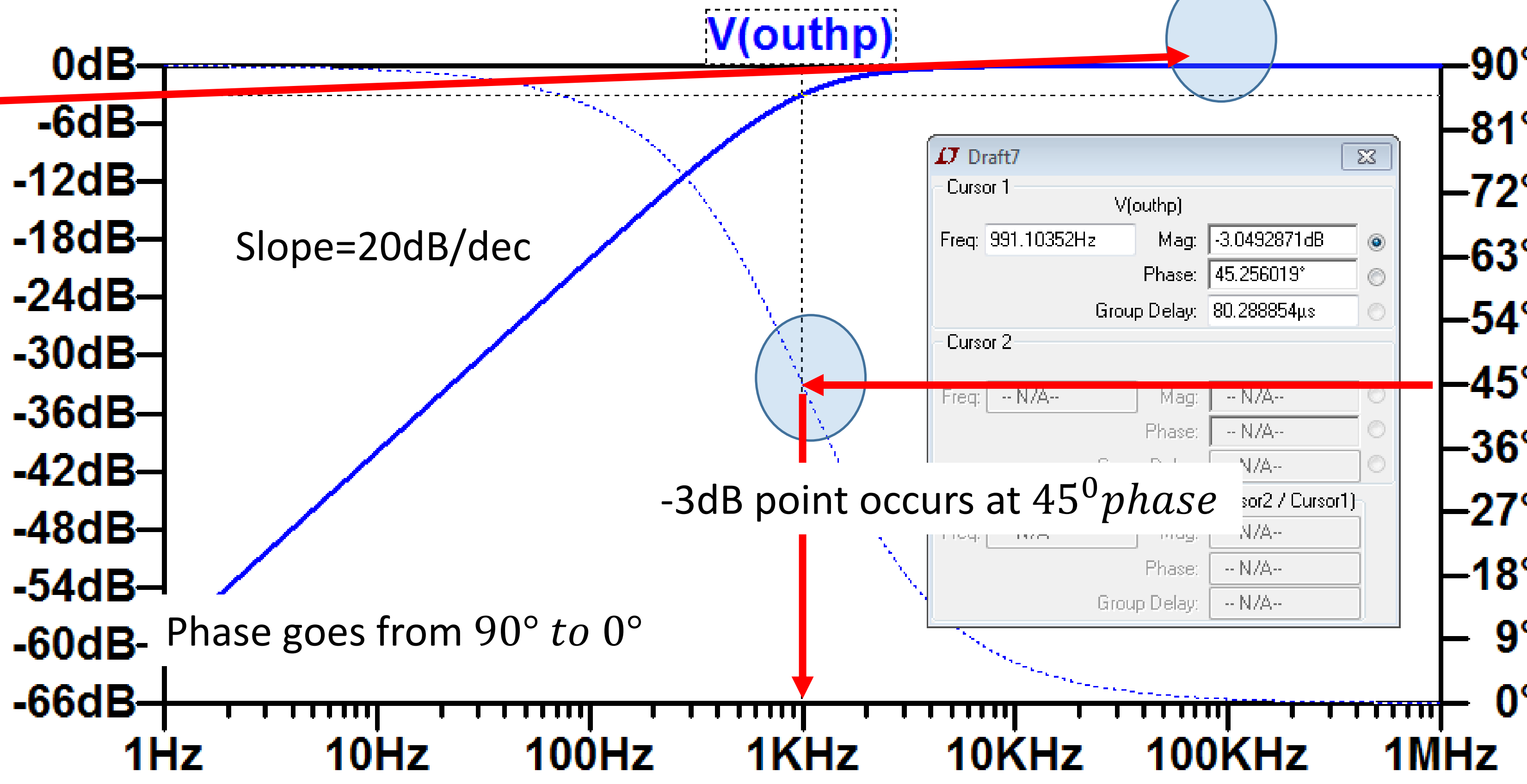
To find ω_0, $set\ H(s = j\omega_0) = \frac{j\omega_0 G_{AC}}{j\omega_0+\omega_0} = \frac{jG_{AC}}{j+1} = \frac{G_{AC}\angle 90°}{\sqrt{2}\angle 45°} = \frac{1}{\sqrt{2}}\angle 45° = -3dB\angle 45°$

To find G_{AC} set s=∞. $H(\infty) = \frac{\infty G_{AC}}{\infty+\omega_0} = G_{AC}$

$$G_{AC} = 0dB = 10^{\frac{0}{20}} = 1$$

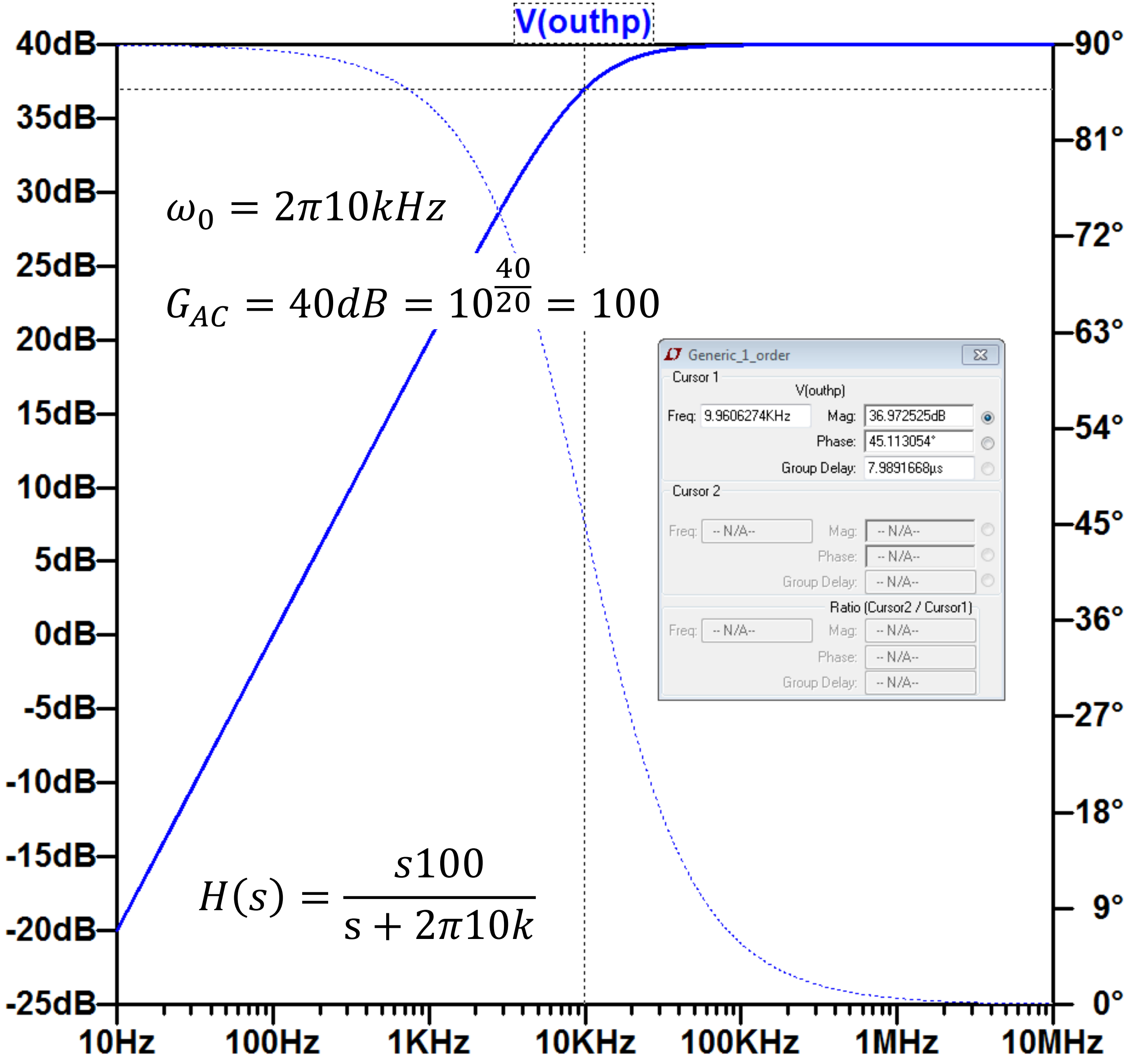
For a first order low pass filter
ω_0 $can\ be\ found\ at + 45°\ phase$
$$\omega_0 = 2 \times \pi \times 1kHz$$
We can then make sure the gain is -3dB down from G_{AC}.

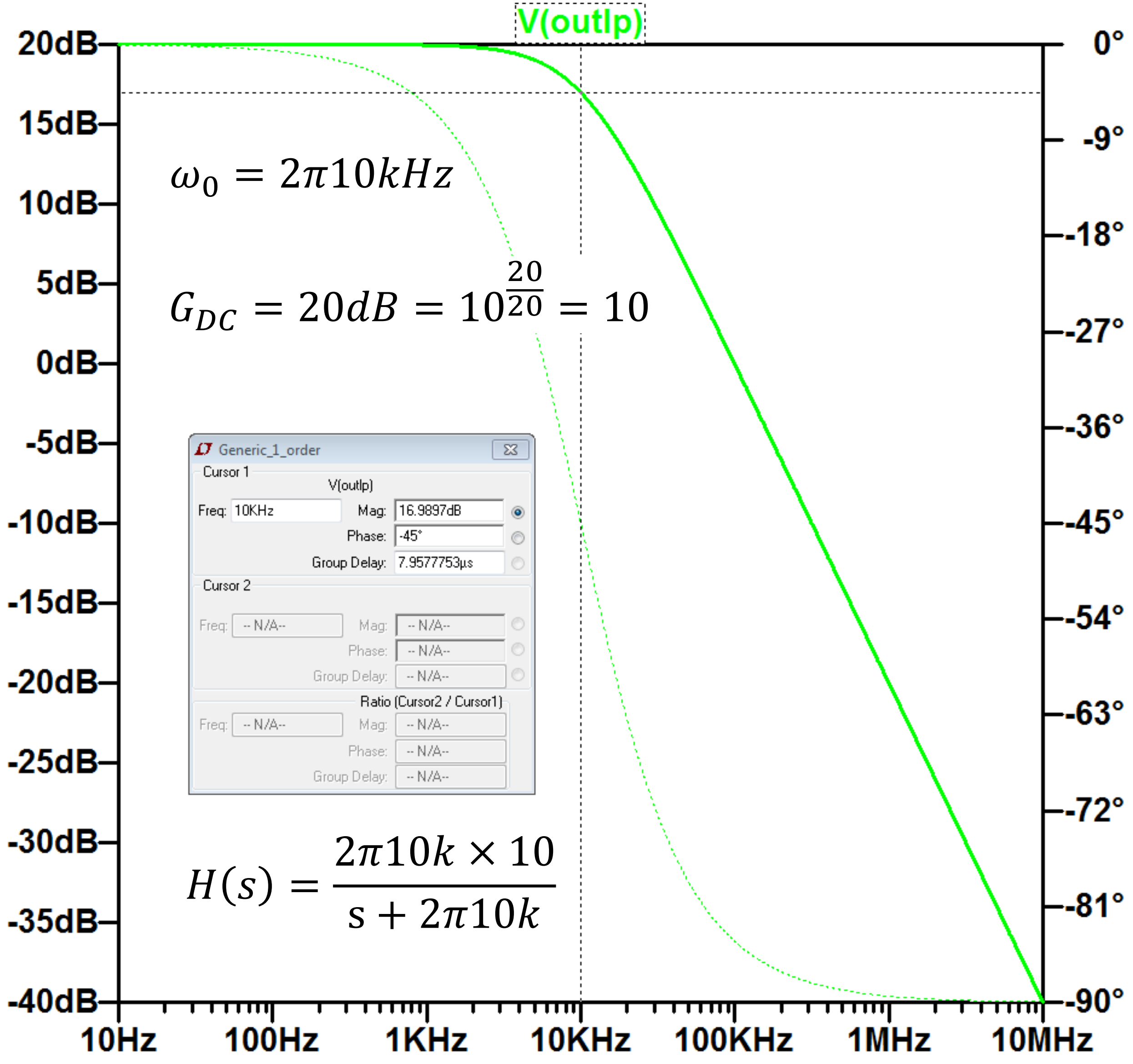
$$H(s) = \frac{s \times 1}{s + 2 \times \pi \times 1k}$$



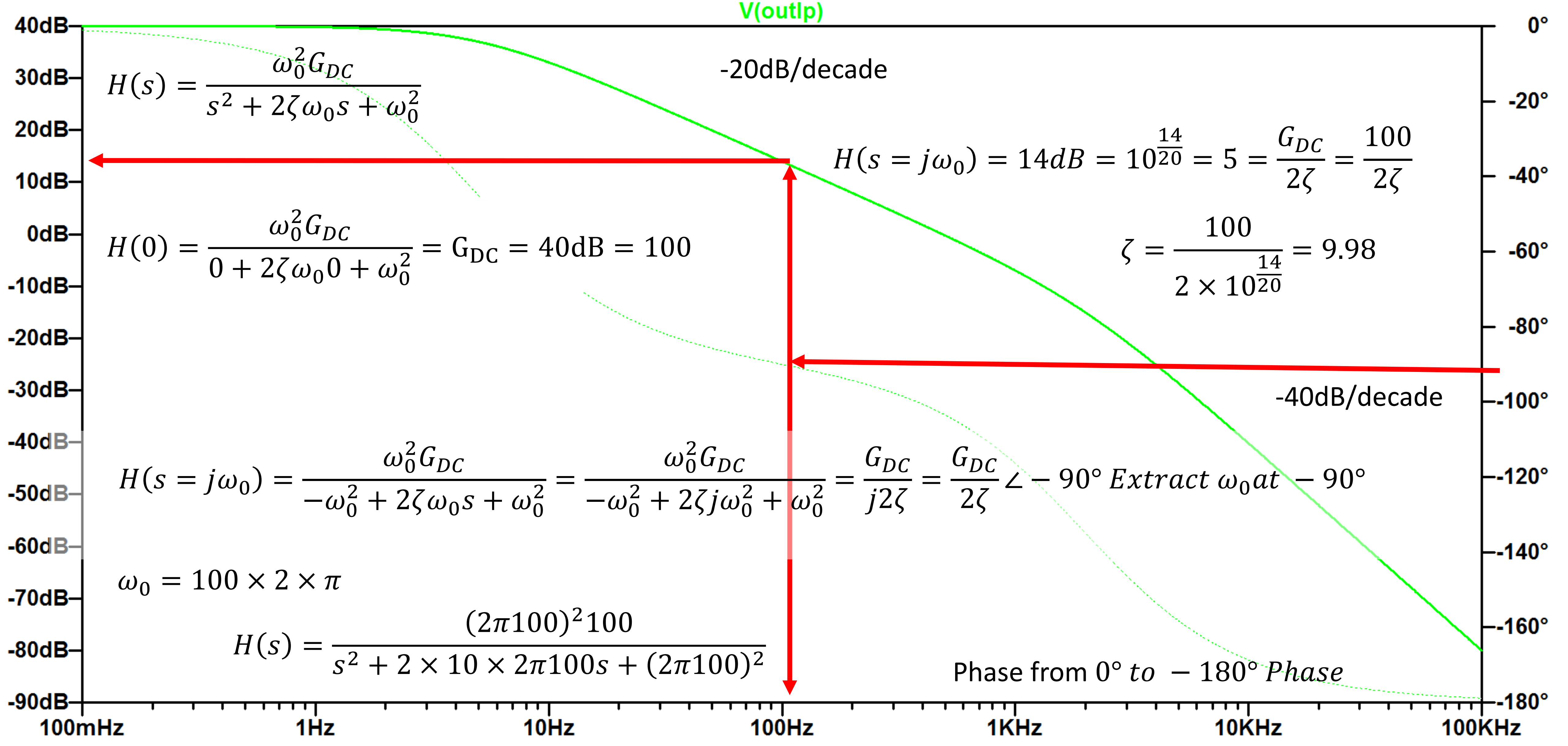
Ltspice File

Sample First order High and Low Pass





Second order Low Pass (Over Damped, two distinct poles)



$$H(s) = \frac{\omega_0^2 G_{DC}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$H(s = j\omega_0) = 14dB = 10^{\frac{14}{20}} = 5 = \frac{G_{DC}}{2\zeta} = \frac{100}{2\zeta}$$

$$H(0) = \frac{\omega_0^2 G_{DC}}{0 + 2\zeta\omega_0 0 + \omega_0^2} = \mathrm{G_{DC}} = 40\mathrm{dB} = 100$$

$$\zeta = \frac{100}{2 \times 10^{\frac{14}{20}}} = 9.98$$

$$H(s = j\omega_0) = \frac{\omega_0^2 G_{DC}}{-\omega_0^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{\omega_0^2 G_{DC}}{-\omega_0^2 + 2\zeta j\omega_0^2 + \omega_0^2} = \frac{G_{DC}}{j2\zeta} = \frac{G_{DC}}{2\zeta}\angle - 90°\ Extract\ \omega_0 at\ - 90°$$

$$\omega_0 = 100 \times 2 \times \pi$$

$$H(s) = \frac{(2\pi 100)^2 100}{s^2 + 2 \times 10 \times 2\pi 100 s + (2\pi 100)^2}$$

Second order Low Pass (critically damped, pair repeated poles)

$$H(s) = \frac{\omega_0^2 G_{DC}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$H(0) = \frac{\omega_0^2 G_{DC}}{0 + 2\zeta\omega_0 0 + \omega_0^2} = \mathrm{G_{DC}} = 40dB = 100$$

$$H(s = j\omega_0) = \frac{G_{DC}}{2\zeta}\angle - 90° \; Extract\ \omega_0 at\ -90°$$

$$\omega_0 = 100 \times 2 \times \pi$$

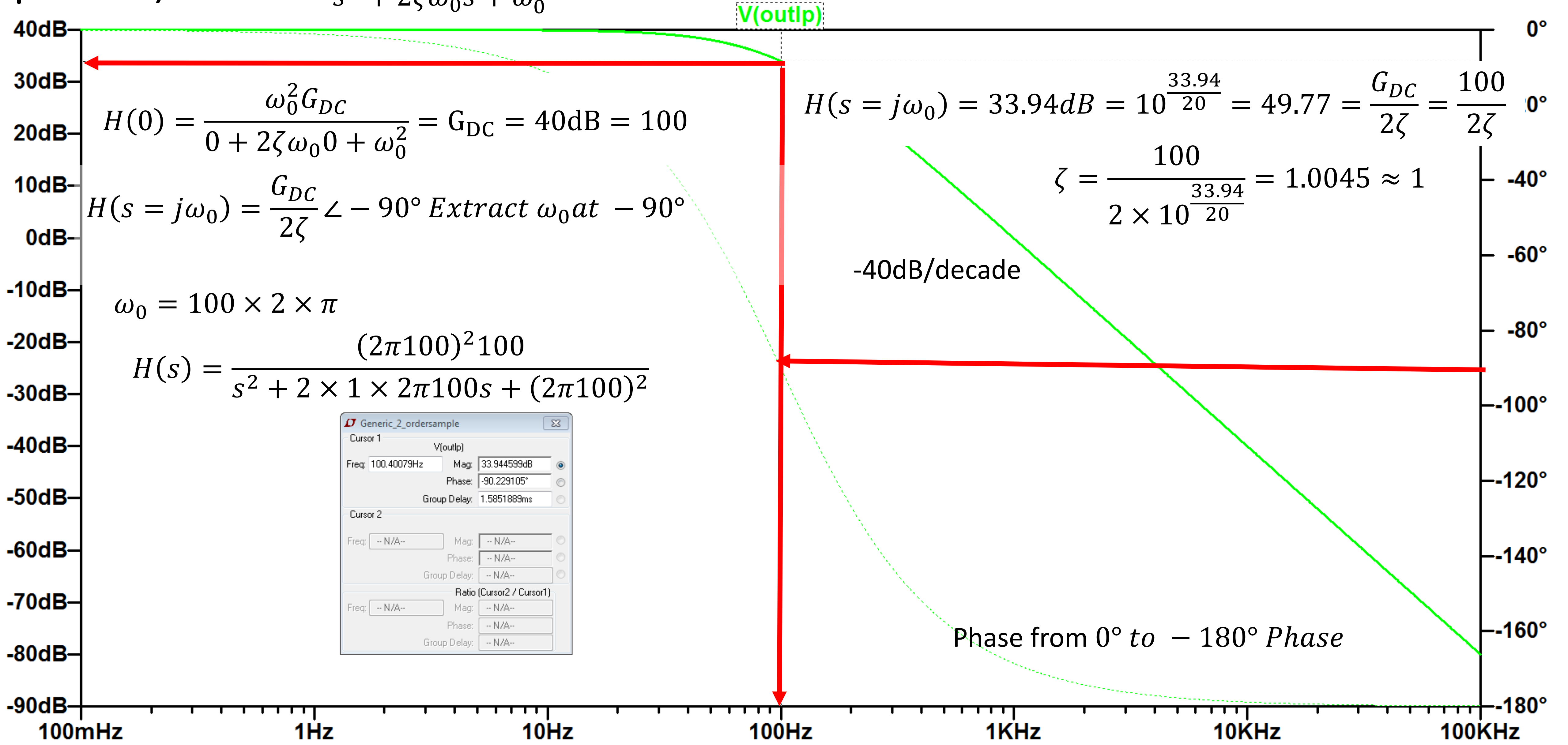
$$H(s) = \frac{(2\pi 100)^2 100}{s^2 + 2 \times 1 \times 2\pi 100 s + (2\pi 100)^2}$$

$$H(s = j\omega_0) = 33.94dB = 10^{\frac{33.94}{20}} = 49.77 = \frac{G_{DC}}{2\zeta} = \frac{100}{2\zeta}$$

$$\zeta = \frac{100}{2 \times 10^{\frac{33.94}{20}}} = 1.0045 \approx 1$$

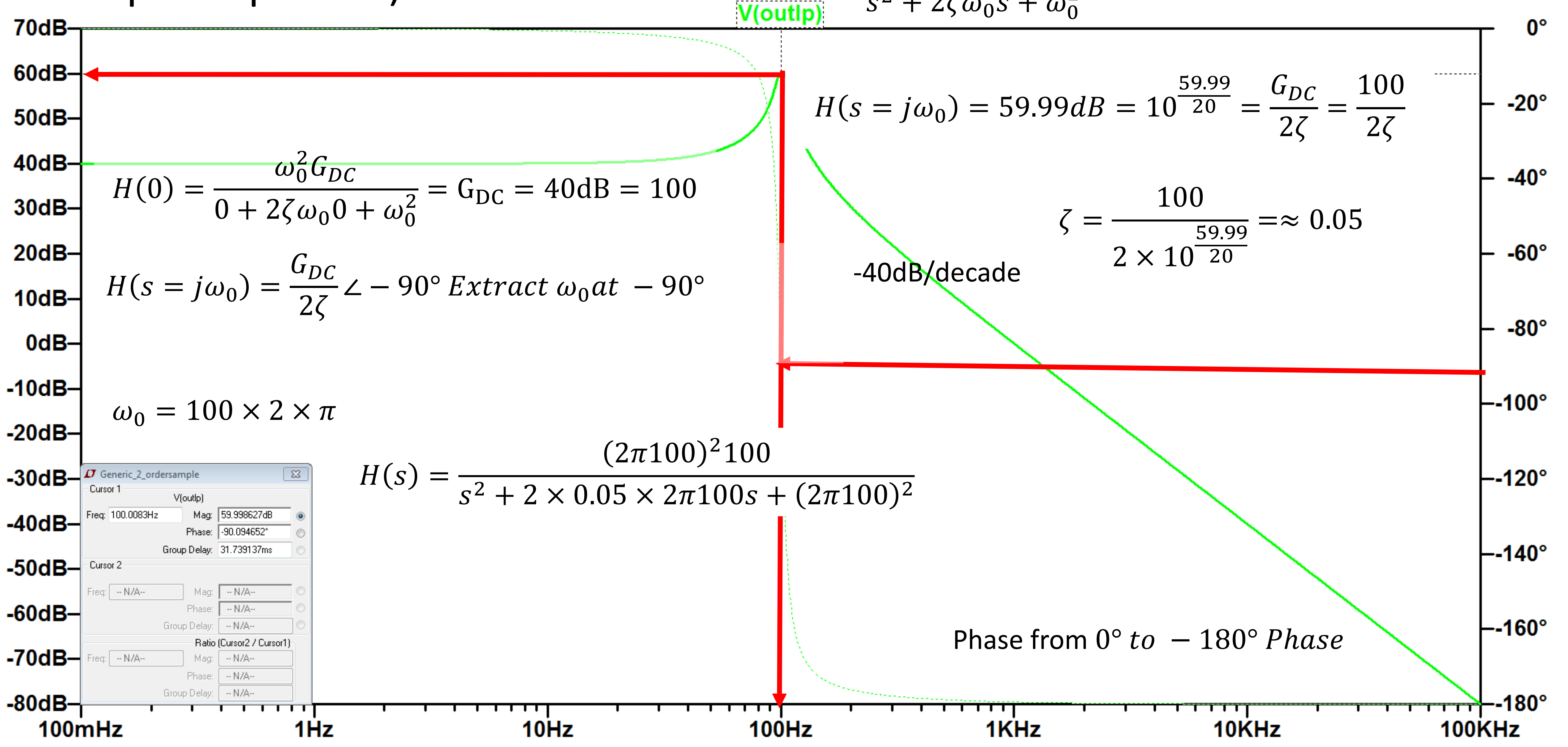
-40dB/decade

Phase from $0°$ *to* $-180°$ *Phase*



Second order Low Pass (underdamped damped, pair complex poles)

$$H(s) = \frac{\omega_0^2 G_{DC}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$



$$H(s = j\omega_0) = 59.99dB = 10^{\frac{59.99}{20}} = \frac{G_{DC}}{2\zeta} = \frac{100}{2\zeta}$$

$$H(0) = \frac{\omega_0^2 G_{DC}}{0 + 2\zeta\omega_0 0 + \omega_0^2} = \mathrm{G_{DC}} = 40\mathrm{dB} = 100$$

$$\zeta = \frac{100}{2 \times 10^{\frac{59.99}{20}}} =\approx 0.05$$

$$H(s = j\omega_0) = \frac{G_{DC}}{2\zeta} \angle -90° \; Extract\ \omega_0 at\ -90°$$

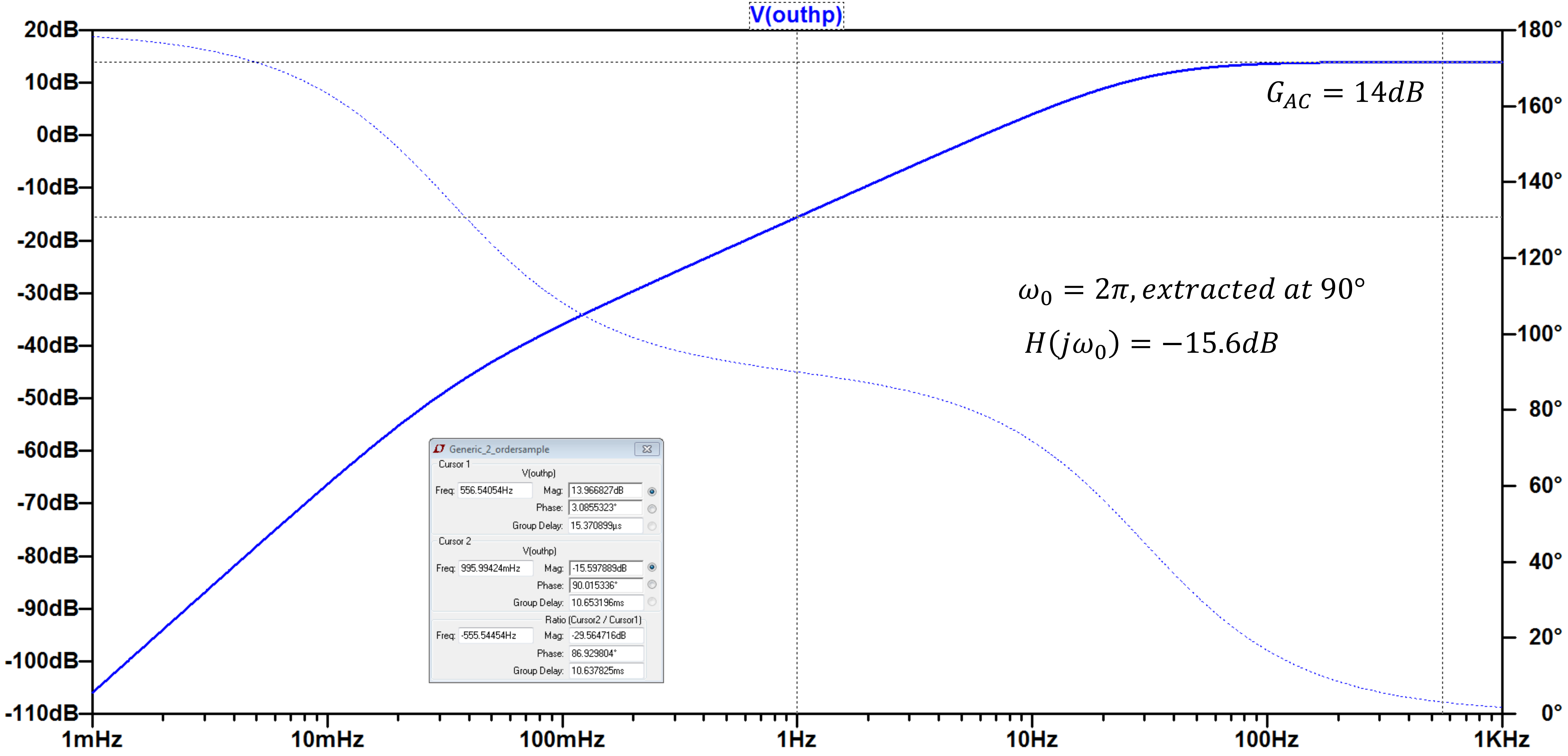
-40dB/decade

$$\omega_0 = 100 \times 2 \times \pi$$

$$H(s) = \frac{(2\pi 100)^2 100}{s^2 + 2 \times 0.05 \times 2\pi 100 s + (2\pi 100)^2}$$

Phase from $0°\ to\ -180°\ Phase$

Second order High Pass (Over damped, pair of real poles)



Second order High Pass (Over damped, pair of real poles)

$$H(s) = \frac{s^2 G_{AC}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$H(\infty) = \frac{s^2 G_{AC}}{s^2} = G_{AC} = 14dB \approx 5$$

$$H(j\omega_0) = -16dB = \frac{-\omega_0{}^2 G_{AC}}{-\omega_0{}^2 + j2\zeta\omega_0{}^2 + \omega_0^2} = \frac{-G_{AC}}{+j2\zeta} = \frac{-G_{AC}}{2\zeta\angle 90°} = \frac{G_{AC}\angle -180°}{2\zeta\angle 90°} = \frac{G_{AC}\angle 180°}{2\zeta\angle 90°}\frac{G_{AC}\angle 90°}{2\zeta}$$

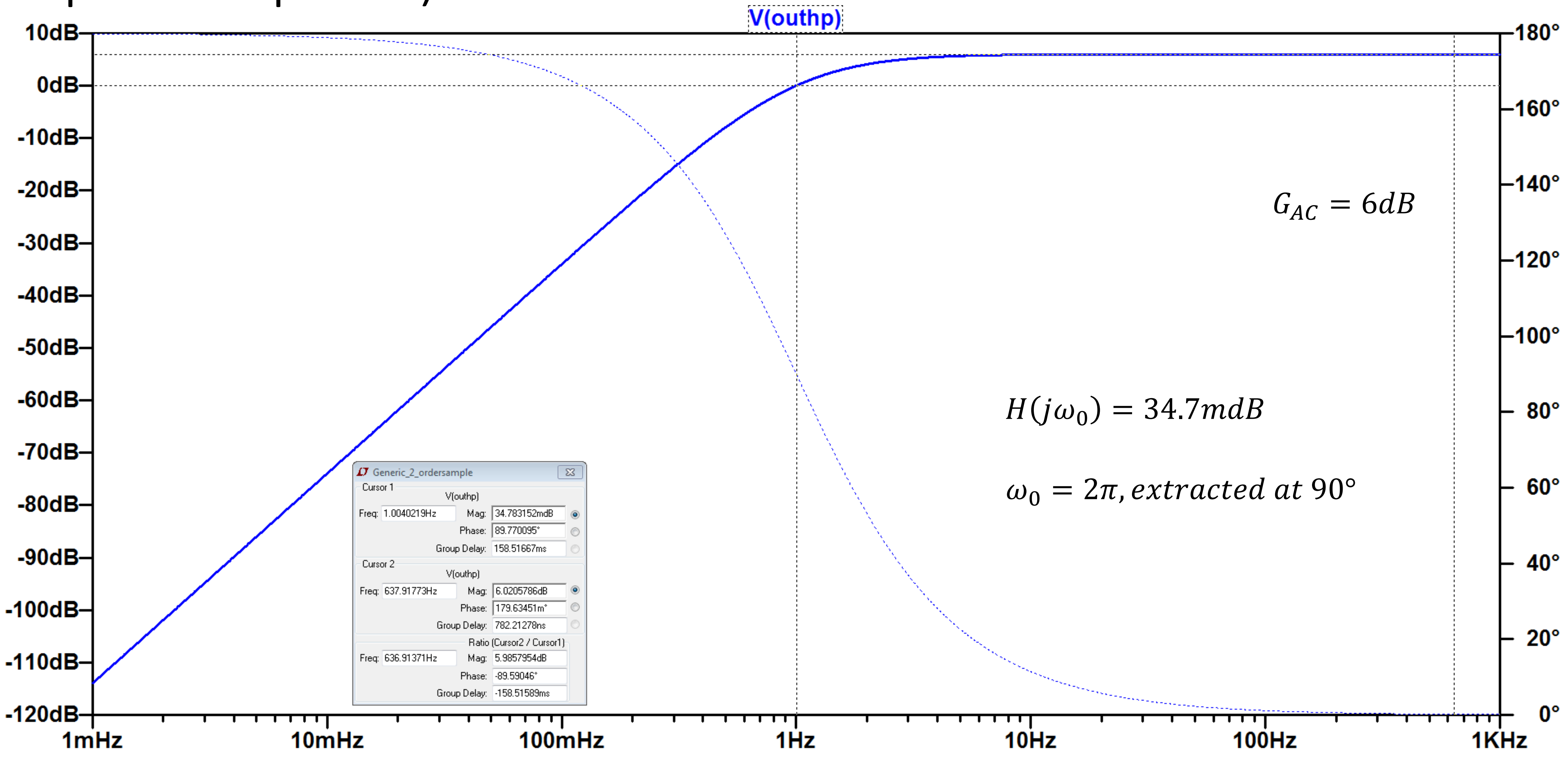
$$\zeta = \frac{G_{AC}}{2\times 10^{-\frac{15.6}{20}}} = \frac{10^{\frac{14}{20}}}{2\times 10^{\frac{-15.6}{20}}} \approx 15$$

$Extract\ \omega_0\ at\ 90°.$

$\omega_0 = 2\pi, extracted\ at\ 90°$

$$H(s) = \frac{s^2 5}{s^2 + 2\times 15\times 2\pi s + (2\pi)^2}$$

Second order High Pass (critically damped, pair of repeated poles)



Second order High Pass (critically damped, pair of repeated poles)

$$H(s) = \frac{s^2 G_{AC}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$H(\infty) = \frac{s^2 G_{AC}}{s^2} = G_{AC} = 6dB \approx 2$$

$$H(j\omega_0) = -34mdB = \frac{-\omega_0{}^2 G_{AC}}{-\omega_0{}^2 + j2\zeta\omega_0{}^2 + \omega_0^2} = \frac{-G_{AC}}{+j2\zeta} = \frac{-G_{AC}}{2\zeta\angle 90°} = \frac{G_{AC}\angle -180°}{2\zeta\angle 90°} = \frac{G_{AC}\angle 180°}{2\zeta\angle 90°}\frac{G_{AC}\angle 90°}{2\zeta}$$

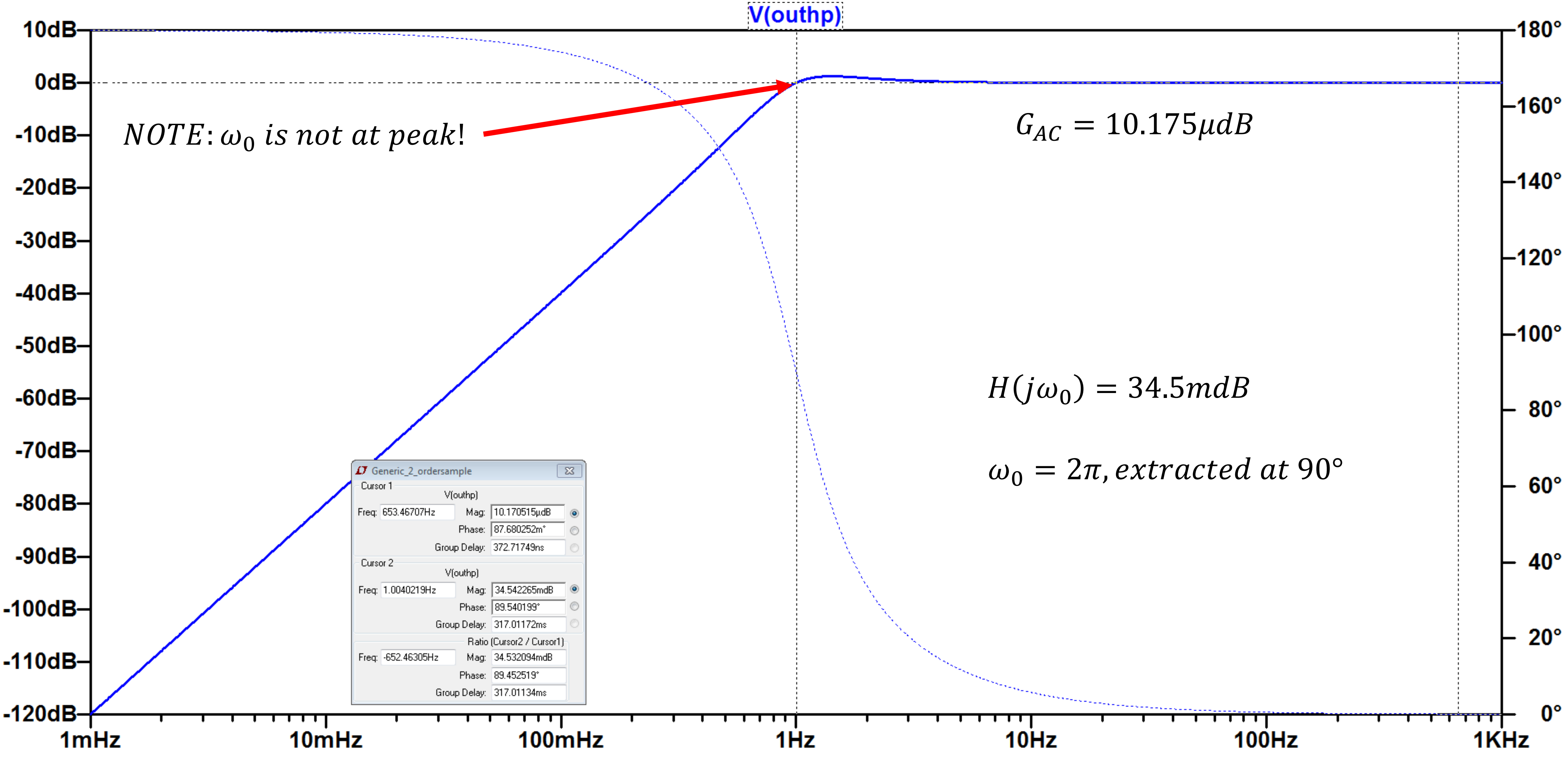
$$\zeta = \frac{G_{AC}}{2 \times 10^{-\frac{34m}{20}}} = \frac{10^{\frac{6}{20}}}{2 \times 10^{\frac{34m}{20}}} \approx 1$$

$Extract\ \omega_0\ at\ 90°.$

$\omega_0 = 2\pi, extracted\ at\ 90°$

$$H(s) = \frac{s^2 2}{s^2 + 2 \times 1 \times 2\pi s + (2\pi)^2}$$

Second order High Pass (under damped, repair of complex poles)



Second order High Pass (under damped, pair of complex poles)

$$H(s) = \frac{s^2 G_{AC}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

$$H(\infty) = \frac{s^2 G_{AC}}{s^2} = G_{AC} = 10.75\mu dB \approx 1$$

$$H(j\omega_0) = -34.5mdB = \frac{-{\omega_0}^2 G_{AC}}{-{\omega_0}^2 + j2\zeta{\omega_0}^2 + \omega_0^2} = \frac{-G_{AC}}{+j2\zeta} = \frac{-G_{AC}}{2\zeta\angle 90°} = \frac{G_{AC}\angle -180°}{2\zeta\angle 90°} = \frac{G_{AC}\angle 180°}{2\zeta\angle 90°}\frac{G_{AC}\angle 90°}{2\zeta}$$

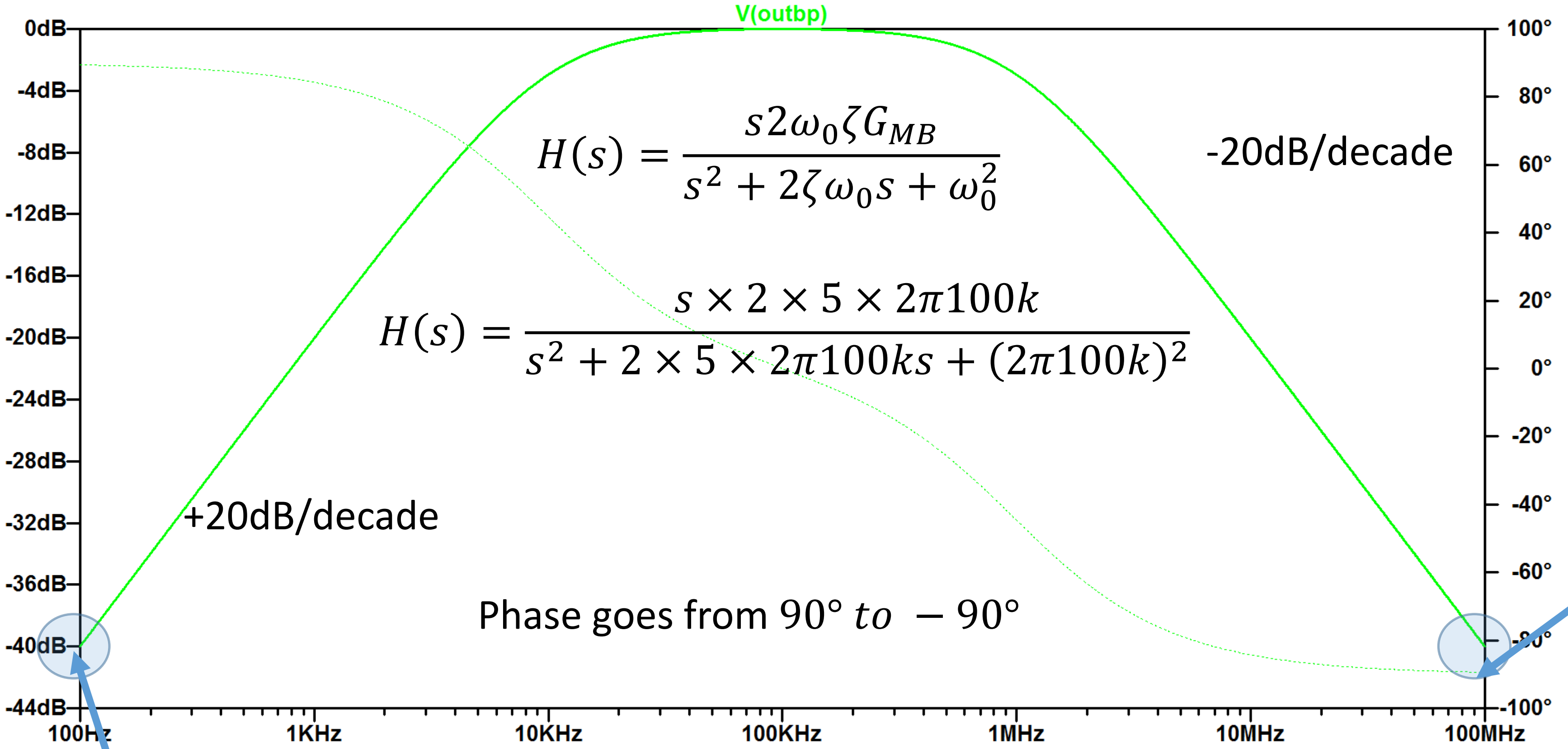
$$\zeta = \frac{G_{AC}}{2 \times 10^{-\frac{34.5m}{20}}} = \frac{10^{\frac{10.75\mu}{20}}}{2 \times 10^{\frac{34.5m}{20}}} \approx \frac{1}{2}$$

$Extract\ \omega_0\ at\ 90°.$

$\omega_0 = 2\pi, extracted\ at\ 90°$

$$H(s) = \frac{s^2}{s^2 + 1 \times 2\pi s + (2\pi)^2}$$

Band Pass (Has to be second order minimum)



$$H(s) = \frac{s2\omega_0\zeta G_{MB}}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$

-20dB/decade

$$H(s) = \frac{s \times 2 \times 5 \times 2\pi 100k}{s^2 + 2 \times 5 \times 2\pi 100ks + (2\pi 100k)^2}$$

+20dB/decade

Phase goes from $90°\ to\ -90°$

$$H(j\omega_0) = \frac{j\omega_0^2 2\zeta G_{MB}}{-\omega_0^2 + j2\zeta\omega_0^2 s + \omega_0^2}$$

$$H(j\omega_0) = \frac{j2\zeta G_{MB}}{j2\zeta} = G_{MB}\angle 0° = 1$$

We extract $\omega_0\ at\ 0°\ phase$

$$\omega_0 = 2\pi 100kHz$$

Extract when phase close to $-90°$

$$H(s \gg \omega_0) = \frac{s\omega_0\zeta G_{MB}}{s^2} = \frac{\omega_0\zeta G_{MB}}{s}$$

$$\zeta = \frac{H(s \gg \omega_0)j\omega}{\omega_0 G_{MB}} = 5$$

$$\zeta = \frac{10^{-\frac{40}{20}}\angle -90^o 2\pi 100M\angle 90^o}{2\pi 100k \times 1} = 5$$

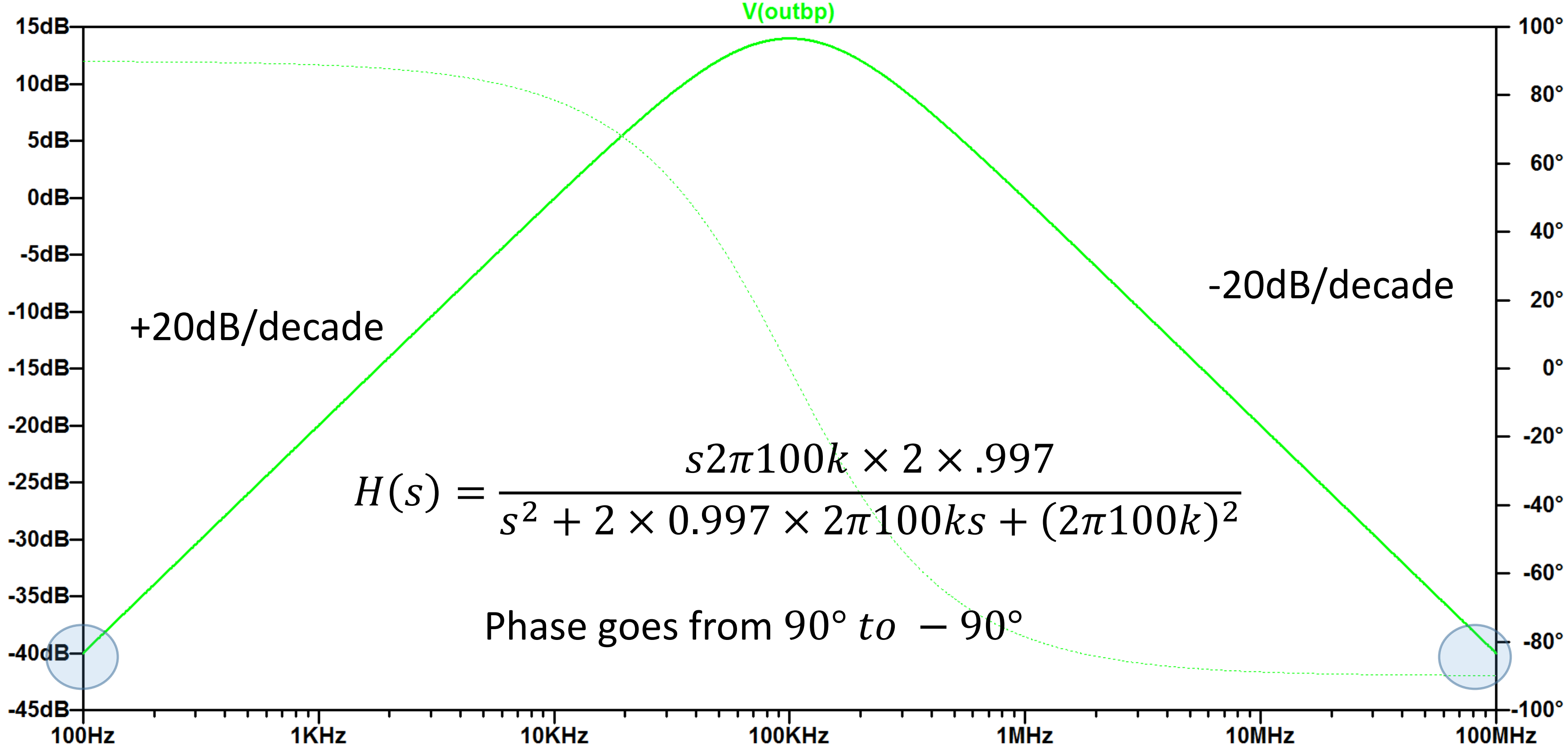
We can find ζ by finding $H(s \ll \omega_0) or\ H(s \gg \omega_0)$

$$H(s \ll \omega_0) = \frac{s2\omega_0\zeta G_{MB}}{s^2 + 2\zeta\omega_0 s + \omega_0^2} = \frac{s2\omega_0\zeta G_{MB}}{\omega_0^2} = \frac{s2\zeta G_{MB}}{\omega_0}$$

Extract when phase close to $90°$ $\zeta = \frac{H(s \ll \omega_0)\omega_o}{j\omega G_{MB} 2} = \frac{10^{-\frac{40}{20}}\angle 90° \times 2\pi \times 100k}{2 \times 2\pi \times 100\angle 90^o \times 1} = 5$

Ltspice Link

Band Pass (Has to be second order minimum)



$$H(s) = \frac{s2\pi 100k \times 2 \times .997}{s^2 + 2 \times 0.997 \times 2\pi 100ks + (2\pi 100k)^2}$$

$$H(j\omega_0) = \frac{j\omega_0^2 2\zeta G_{MB}}{-\omega_0^2 + j2\zeta\omega_0^2 s + \omega_0^2}$$

$$H(j\omega_0) = G_{MB}\angle 0° = 10^{\frac{14}{20}} = 5.001$$

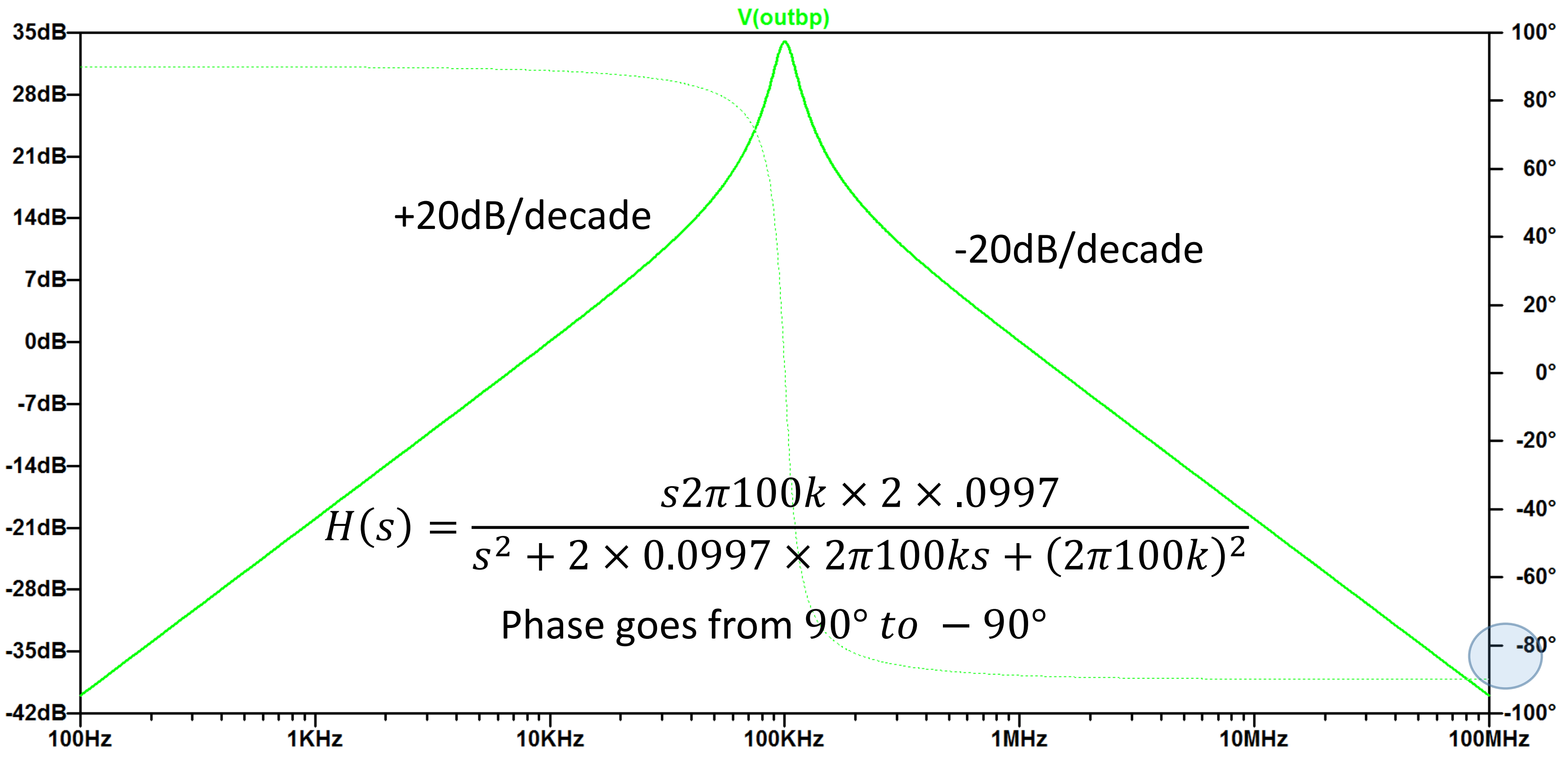
We extract ω_0 *at* $0°$ *phase*

$$\omega_0 = 2\pi 100kHz$$

$$\zeta = \frac{10^{-\frac{40}{20}} \times 2\pi \times 100k}{2 \times 2\pi \times 100 \times 5.011} = .997$$

$$\zeta = \frac{10^{-\frac{40}{20}} \times 2\pi 100M}{2\pi \times 100k \times 5.011} = .997$$

Band Pass (Has to be second order minimum)



$$H(s) = \frac{s2\pi 100k \times 2 \times .0997}{s^2 + 2 \times 0.0997 \times 2\pi 100ks + (2\pi 100k)^2}$$

Phase goes from $90°\ to\ -90°$

$$H(j\omega_0) = \frac{j\omega_0^2 2\zeta G_{MB}}{-\omega_0^2 + j2\zeta\omega_0^2 s + \omega_0^2}$$

$$H(j\omega_0) = G_{MB}\angle 0° = 10^{\frac{34}{20}} = 50.11$$

We extract $\omega_0\ at\ 0°\ phase$

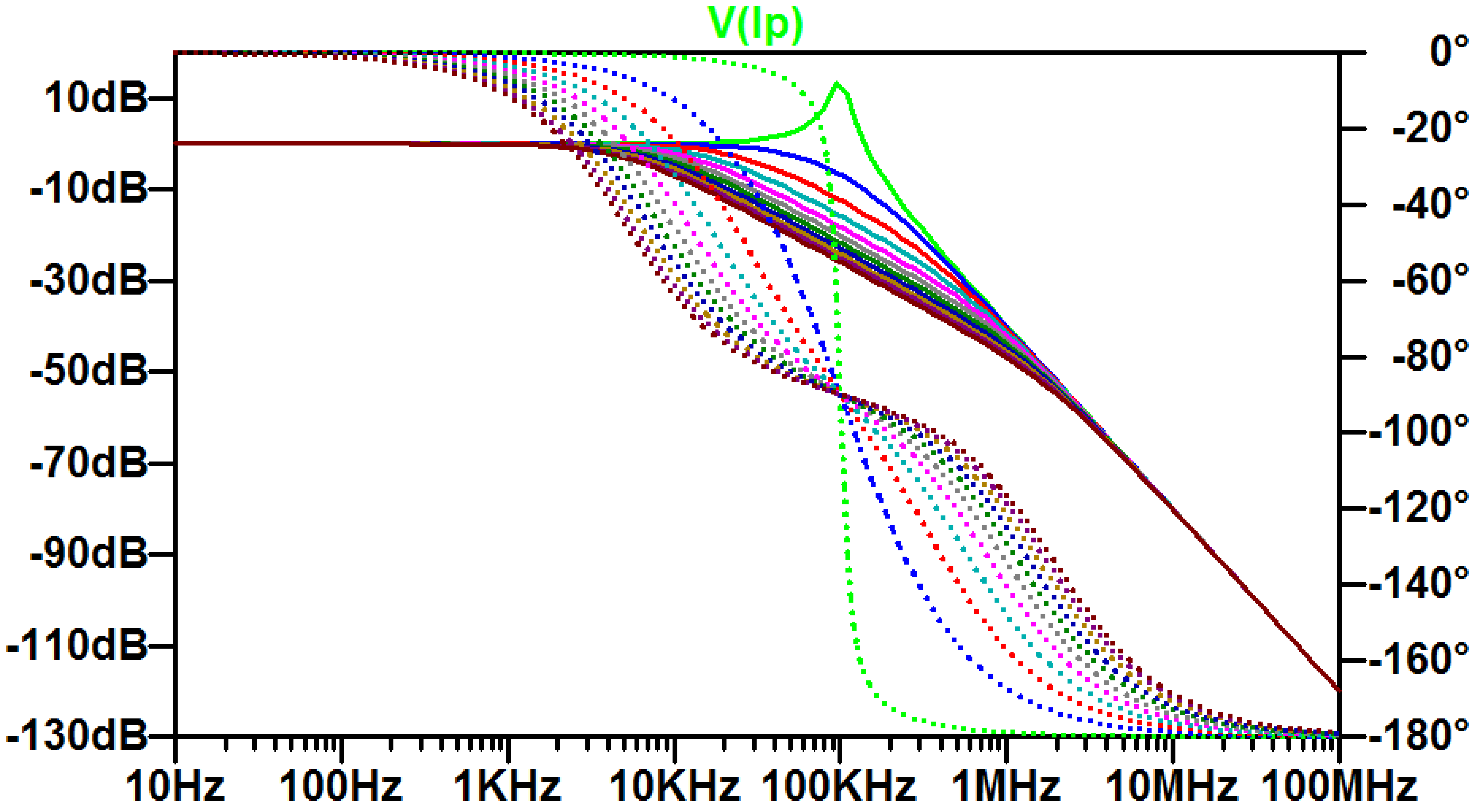
$$\omega_0 = 2\pi 100kHz$$

$$\zeta = \frac{10^{-\frac{40}{20}} \times 2\pi \times 100k}{2 \times 2\pi \times 100 \times 50.11} = .0997$$

$$\zeta = \frac{10^{-\frac{40}{20}} \times 2\pi 100M}{2\pi \times 100k \times 50.11} = .0997$$

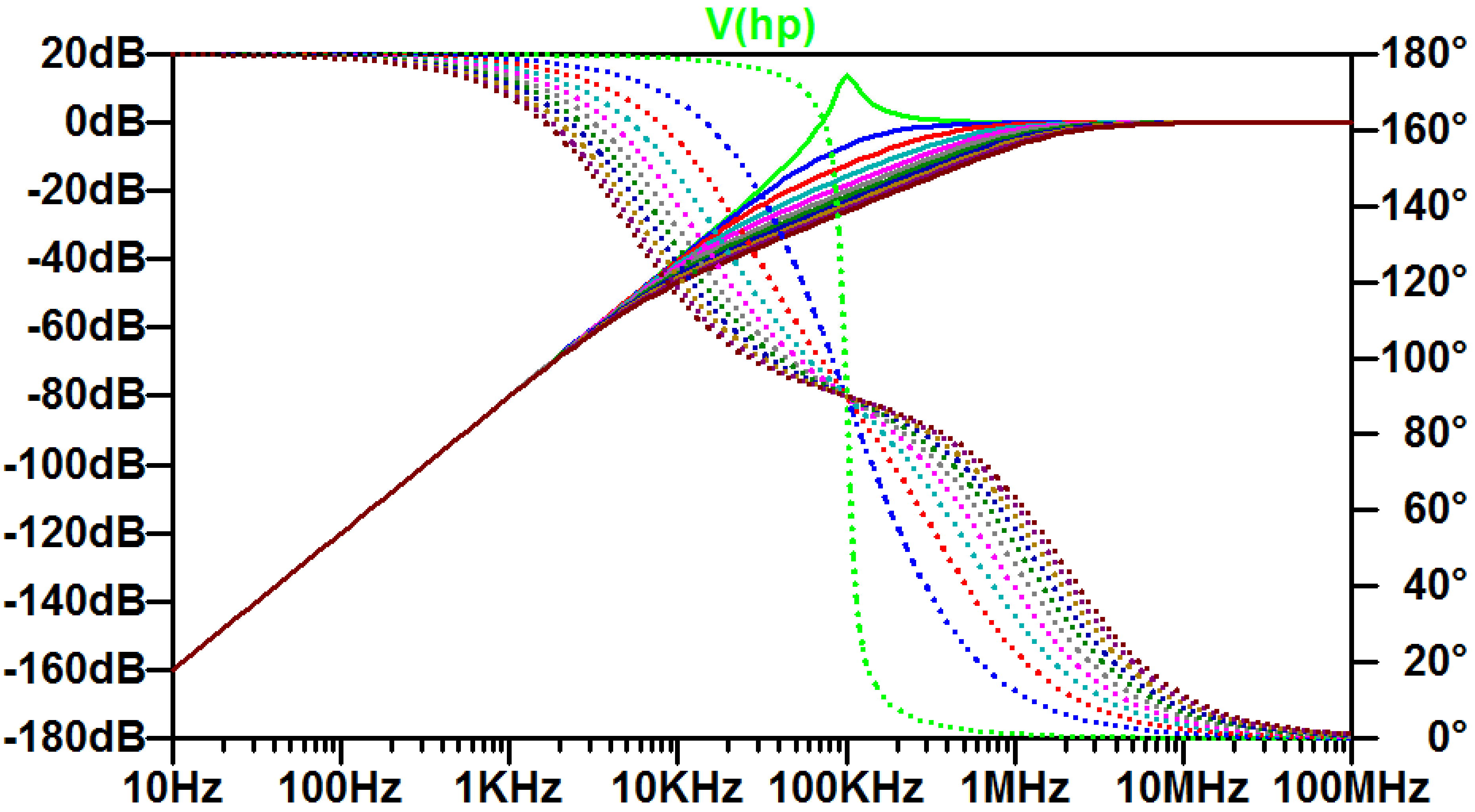
Low pass with ζ varied from .1 to 10

[Link]



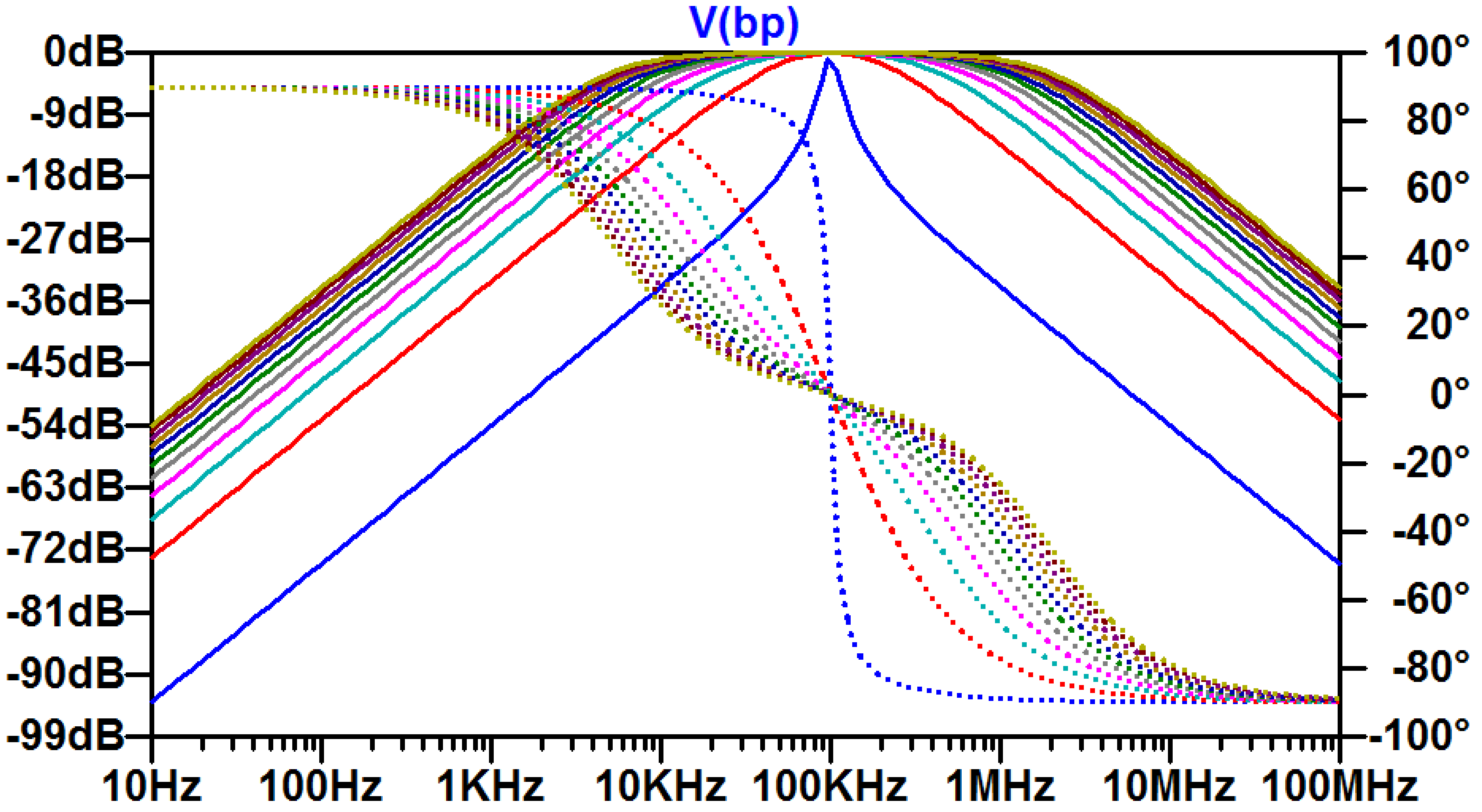
High pass with ζ varied from .1 to 10

[Link]



Band pass with ζ varied from .1 to 10

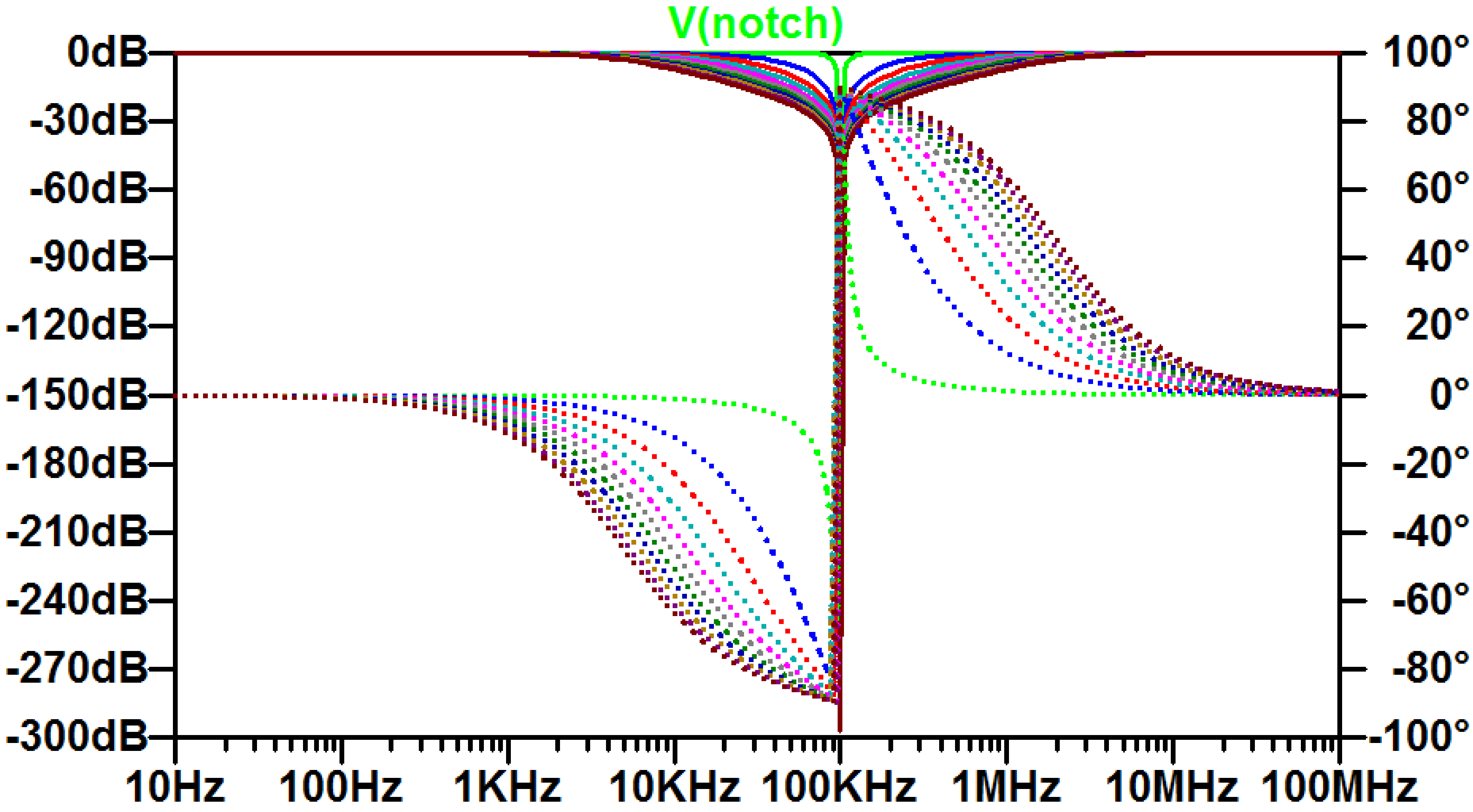
Link



Note: $G_{MB} = 1$ for all Values of ζ in this example.

In this example it is $H(s \ll j\omega)$ that changes

Bandstop (notch)with ζ varied from .1 to 10



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